Laboratoire Kastler Brossel
Physique quantique et applications

ENS
cnrs
SORBONNE UNIVERSITÉ

Continuous variable quantum networks -3

Valentina Parigi

Multimode quantum optics group

Continuous Variables Quantum Complex Networks team

Okinawa School in Physics: From quantum key distribution to the quantum internet (OSP2025)

September 21, 2025 - October 3, 2025

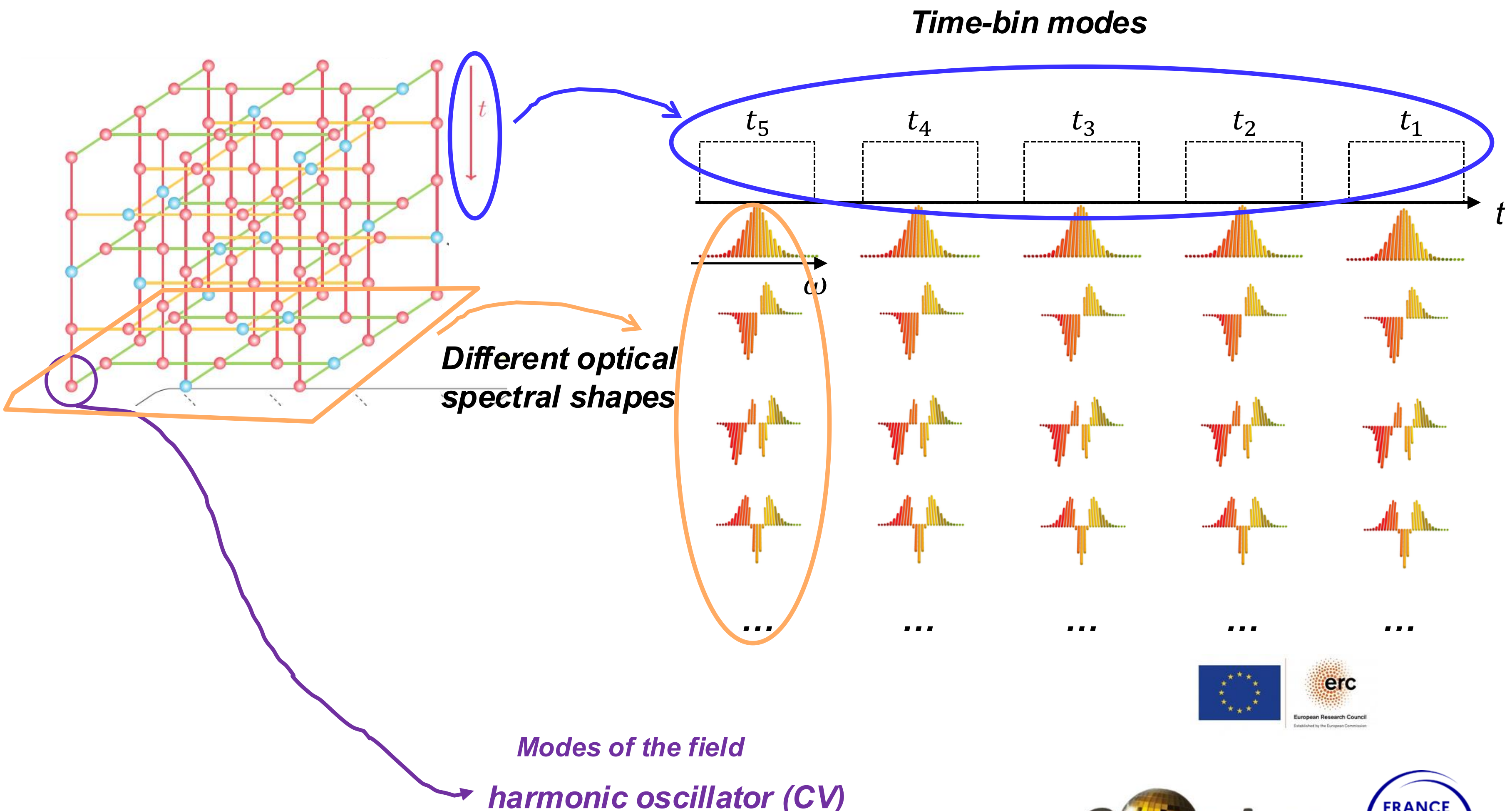
CV cluster states as quantum networks

Cluster states as
Quantum internet
(local area network)

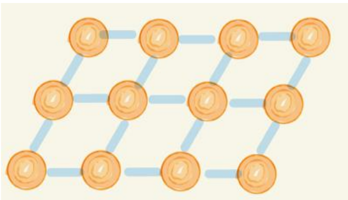
Spectral and time multiplexing

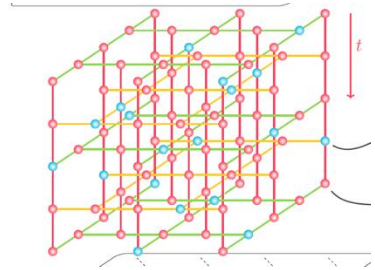
Our strategy

- *Deterministic room-temperature* *generation of large number of Gaussian entangled states*



Telecom setup



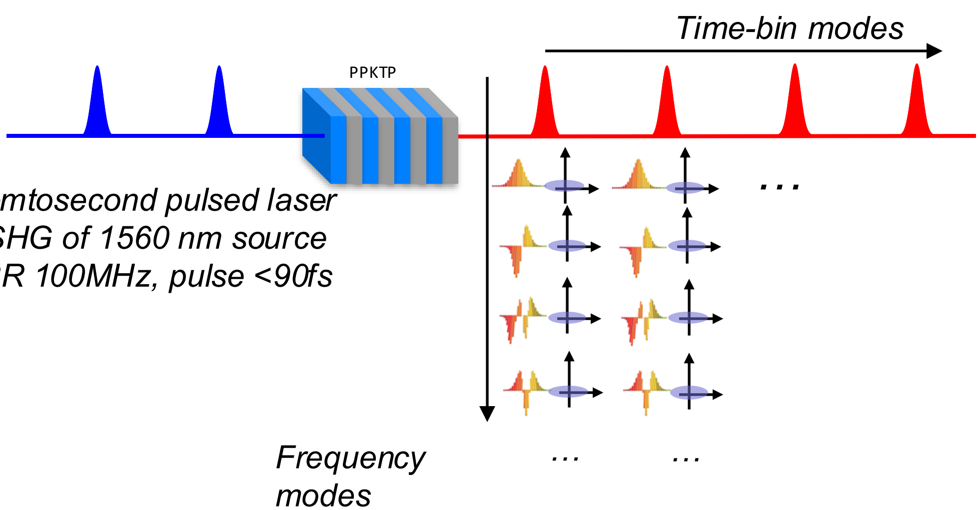


@telecom!

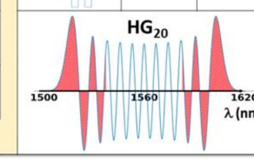
21 modes with more than 2.5 dB of squeezing

Building block

Large number of involved modes:
merging strategy based on optical spectral shape with the one based on time-bin



Hermite-Gauss	Sqz	ASqz		Sqz	ASqz	Flat modes	Sqz	ASqz
0	-1.03	1.39	6	-0.73	1.45	0	-2.66	6.99
1	-0.68	1.31	7	-0.74	1.27	1	-2.43	6.49
2	-0.62	1.16	8	-0.54	1.16	2	-2.32	6.83
3	-0.61	1.27	12	-0.55	1.15	3	-2.01	6.47
4	-0.57	1.41	15	-0.60	1.36			
5	-0.58	1.46	20	-0.53	0.85			



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V. Roman-Rodriguez, B. Brecht, K. Srinivasan, C. Silberhorn, N. Treps, E. Diamanti, V. Parigi, *New J. Phys.* 23 043012 (2021)
V. Roman-Rodriguez, D. Fainsin, G. L. Zanin, N. Treps, E. Diamanti, and V. Parigi Physical Review Research 6, 043113 (2024)

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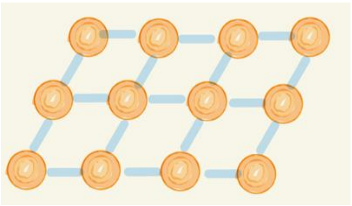

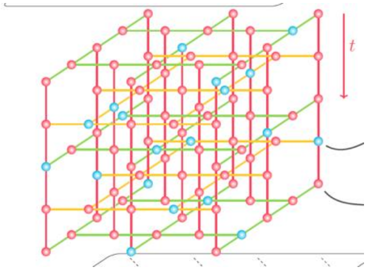

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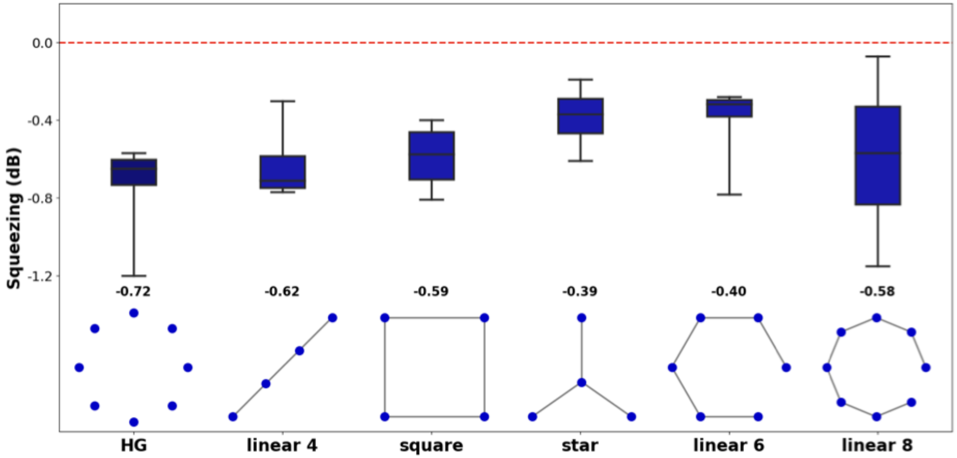

+ Bipartite PPT criteria when measuring In 8 frequency band basis -> all the bossible 127 bipartitions are entangled

+Test : cluster states generation

…

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Telecom setup

LKB

Can we distribute the different nodes (spectral multiplexing) to use as a (small –distances) quantum networks ?

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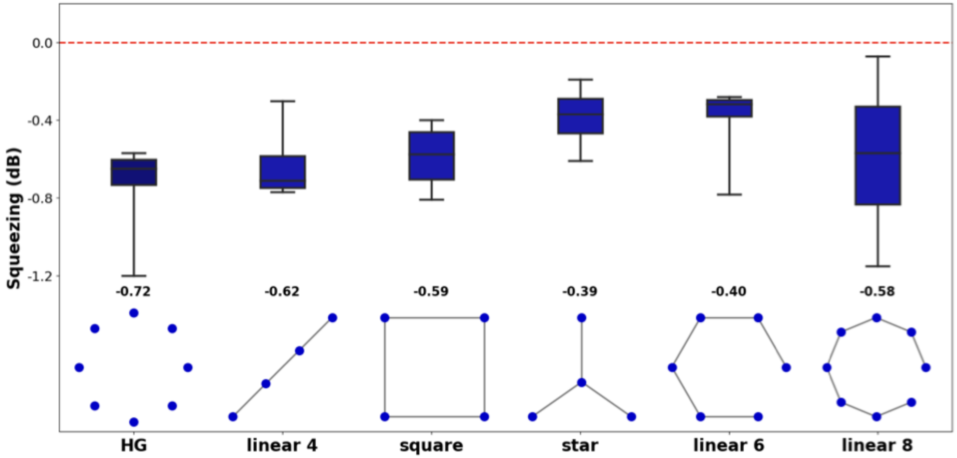

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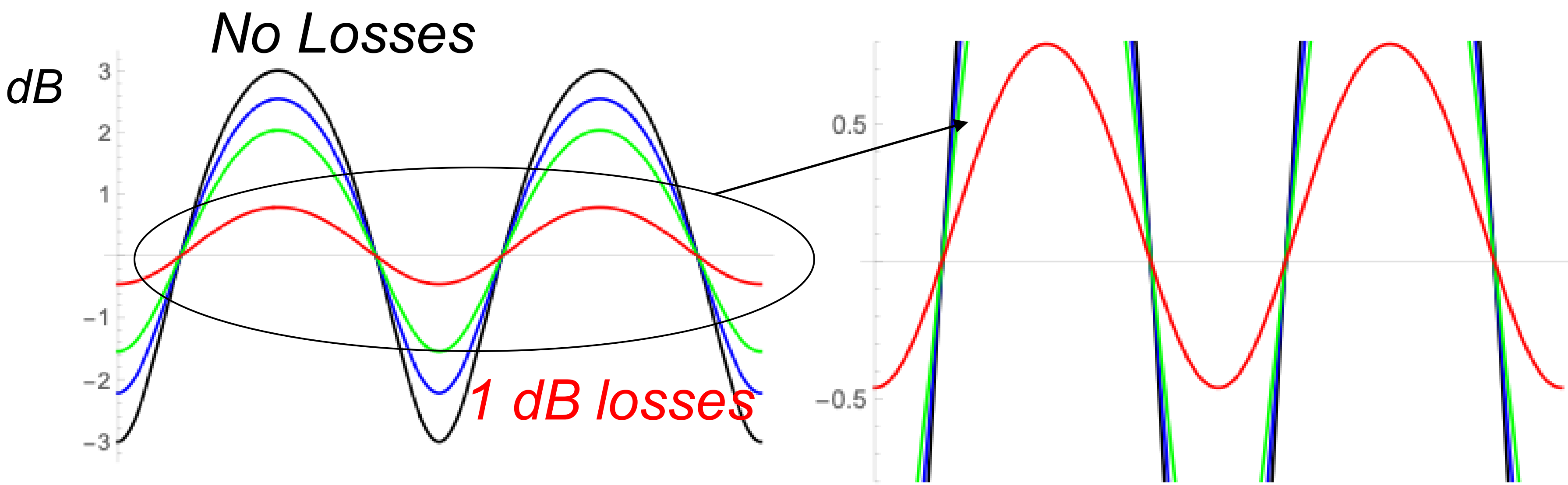
Can we distribute the different nodes (spectral multiplexing) to use as a (small –distances) quantum networks ?

Let's look for a second at the elephant in the room

CV resources are not very much resilient to losses



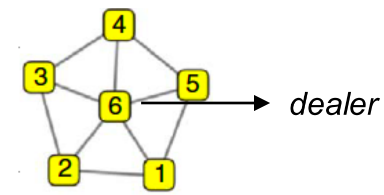
From 3dB of squeezing to less than 0.5, and extra-noise (anti-squeezing)

A quantum communication protocol with cluster at short distance

Quantum communications with clusters

five-partite secret sharing protocol with six mode all-optical quantum graph

A cluster for secret sharing



- Secret can only be retrieved through a collaboration of subsets of the 3 players

- quantum correlations increase both the protocol **security** as well as its retrieval **fidelity** compared to classical resources

Y. Cai, J. Roslund, G. Ferrini, F. Arzani, X. Xu, C. Fabre & N. Treps, Nature Communications 8,15645 (2017)

Quantum communications with clusters

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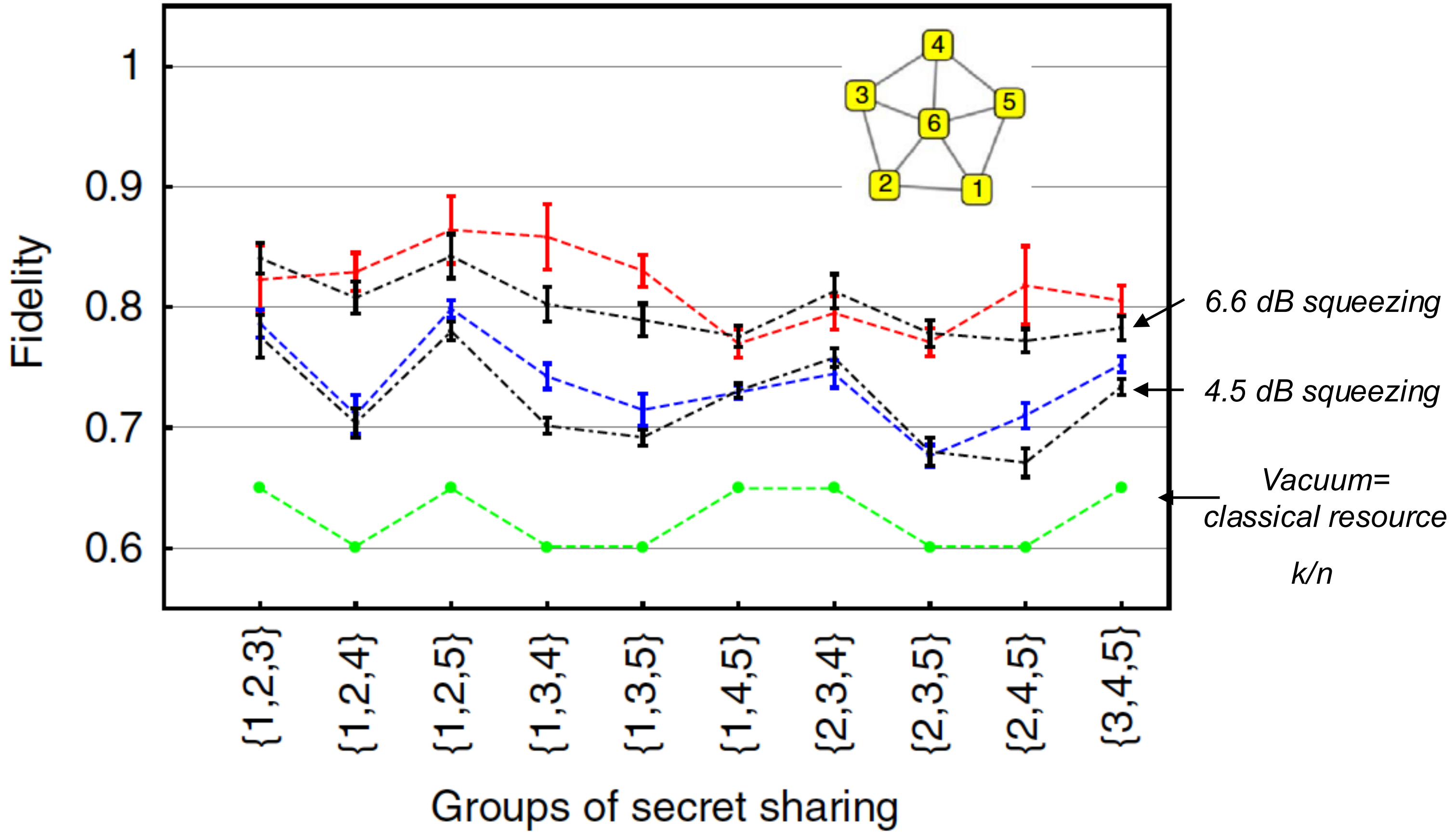

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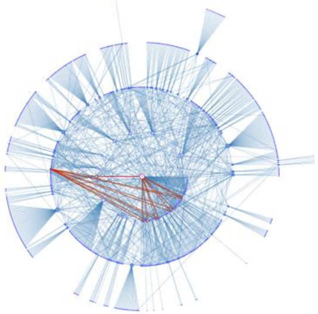
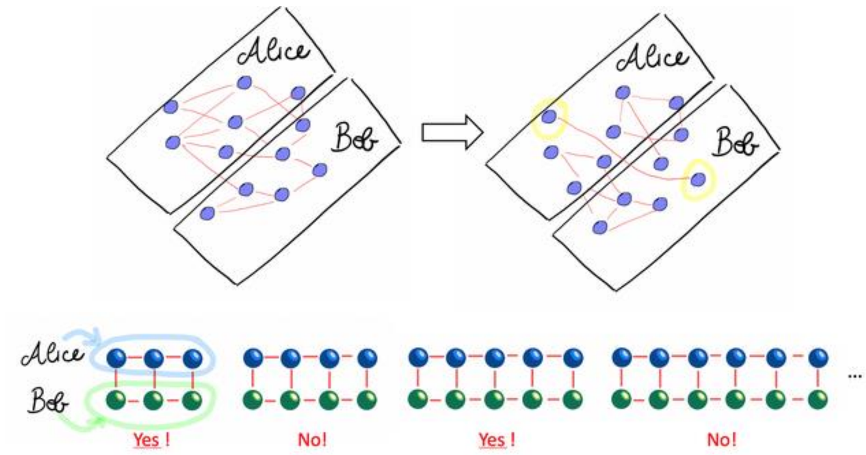
Theory:
Look at very large cluster/graph
With complex-netwoks shape
(entanglement routing)



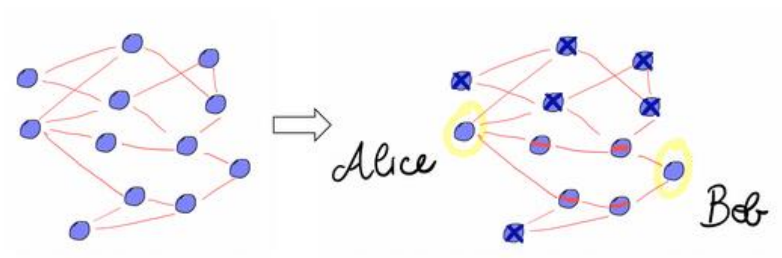
Cost and routing of a graph (cluster)

Idea: in the CV case is easier to prepare large entanglement networks and then reshape

Via linear optics



Via measurements



F. Sansavini and V. Parigi, *Entropy* 22, 26 (2019)
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F. Centrone *et al.* Physical Review A 108 (4), 042615 (2023)

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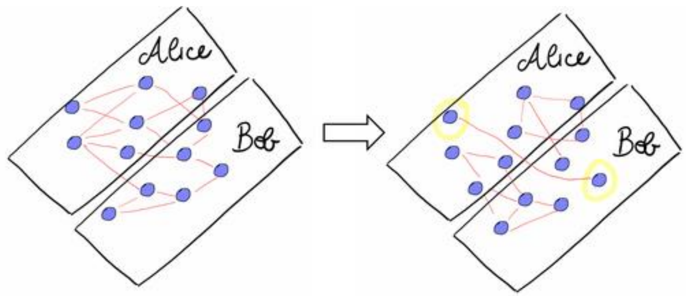

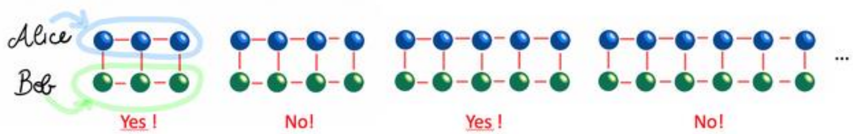

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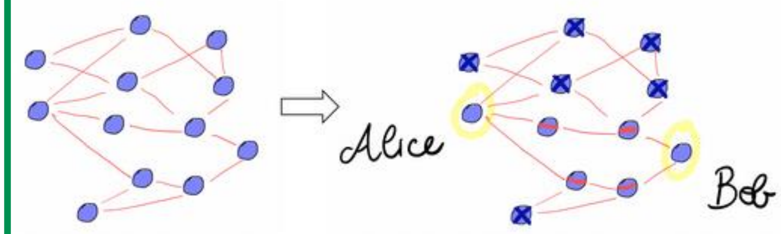

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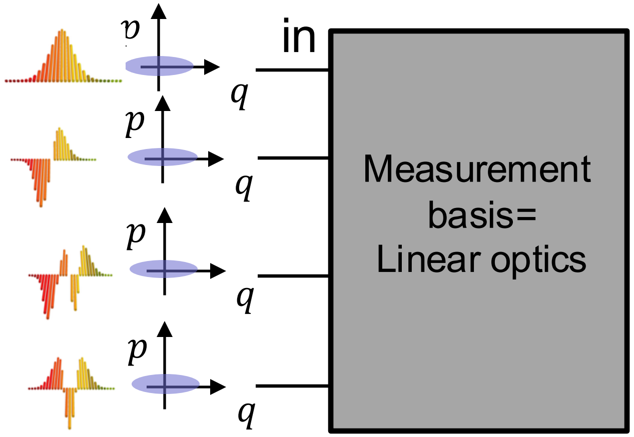

squeezing spectrum

$$\lambda_i^{\pm} = = \frac{1}{2}\left(1 + D_i^2/2 \pm \sqrt{D_i^2 + D_i^4/4}\right)$$

eigenvalues of the adjacency matrix

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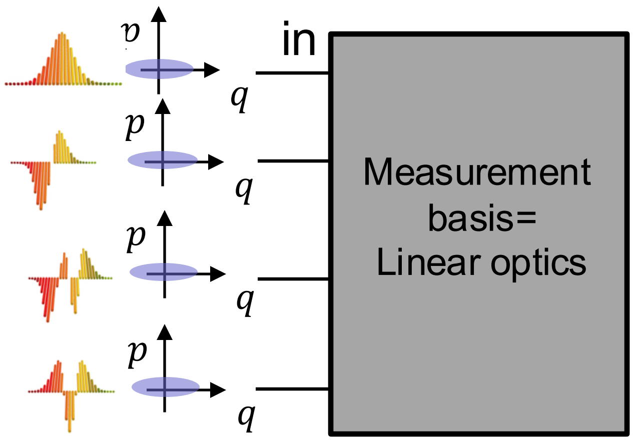

$$\{q_1, q_2, ..q_N, p_1, p_2, ...p_N\} = \{\mathbf{q}, \mathbf{p}\} = \mathbf{X}_b^T$$

$$\Gamma_{j,k} = (1/2)\langle \hat{\xi}_i \hat{\xi}_j + \hat{\xi}_j \hat{\xi}_i \rangle$$

Bloch Messiah decomposition $p_{i,j}$ *or* $q_{i,j}$

$$\mathbf{X}_f = S\mathbf{X}_i$$

$$S = R_1 \Delta^{sq} R_2$$

If $\mathbf{X}_i$ *are the quadratures of the vacuum you can discard R2*

$$\Gamma = (1/2) S S^T$$

squeezing spectrum

$$\lambda_i^{\pm} = = \frac{1}{2}\left(1 + D_i^2/2 \pm \sqrt{D_i^2 + D_i^4/4}\right)$$

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$\lambda_i^{\pm}$: *are the diagonal elements of* Δ

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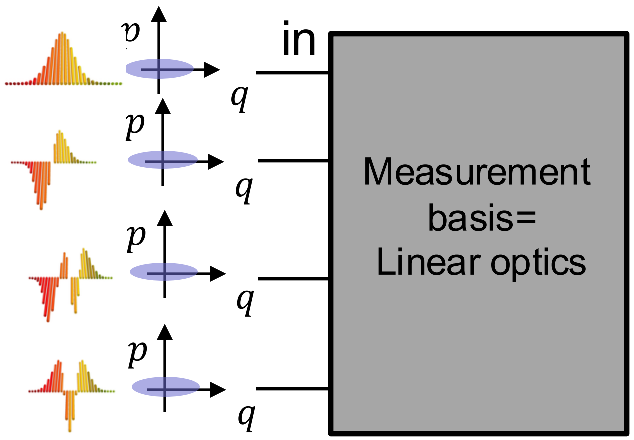

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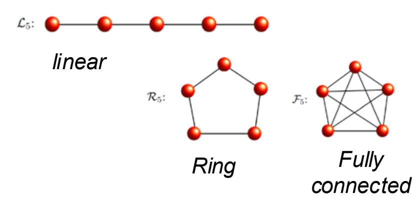
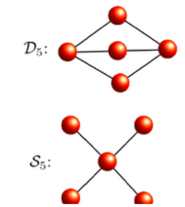
$$D_k(\mathcal{L}_N) = 2\cos\frac{\pi k}{N+1},\ \{k = 1, ..., N\}$$



$$\{D_k(\mathcal{D}_N)\} = \{\sqrt{2N}, 0^{N-2}, -\sqrt{2N}\}.$$



$$\{D_k(\mathcal{S}_N)\} = \{\sqrt{N-1}, 0^{N-2}, -\sqrt{N-1}\},$$

Consequence: the number of independent squeezed modes (squeezers) in their Bloch–Messiah decomposition corresponds to the rank rk(A) of the associated adjacency matrix

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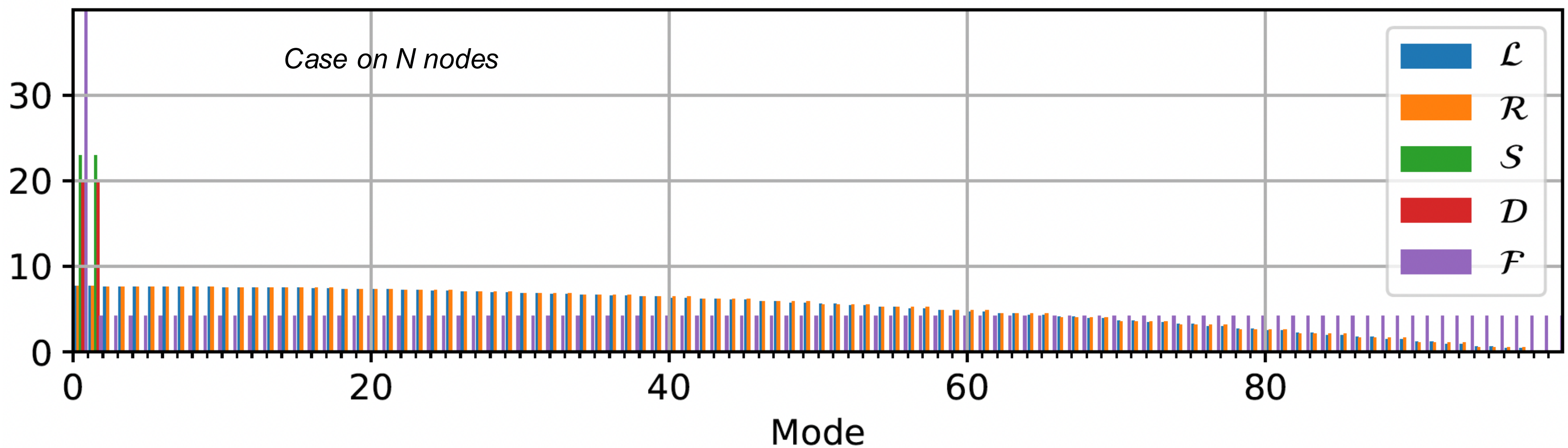

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$$G(\sigma) = \sum_{i=1}^{N} 10 \log_{10} \left(\lambda_i^+ (2\sigma) \right) .$$

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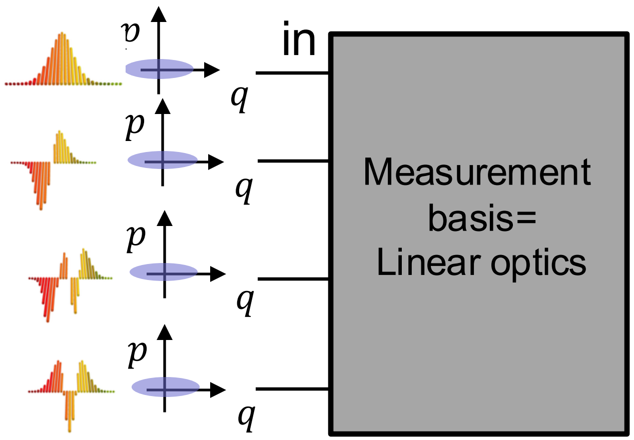

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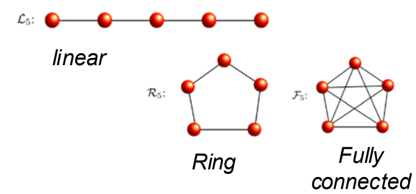

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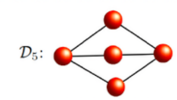

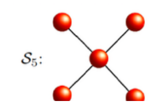

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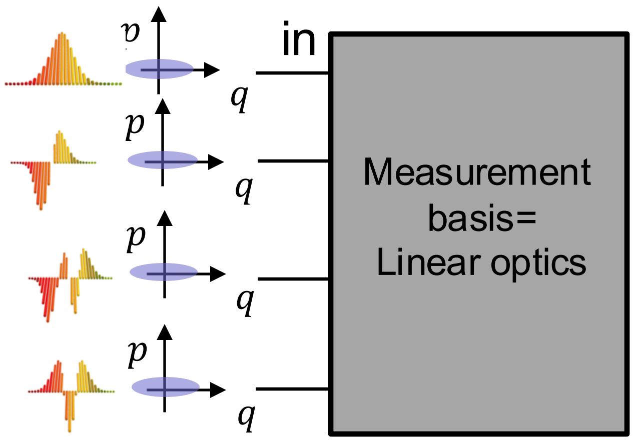

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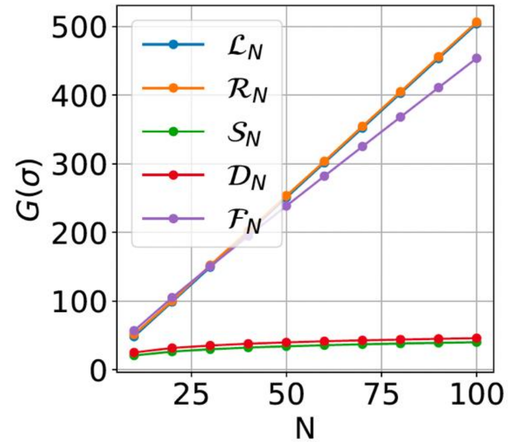
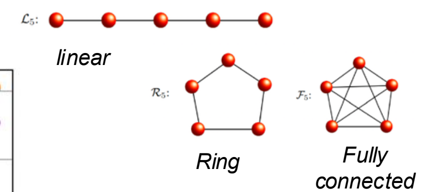
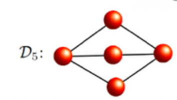
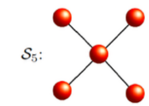




$$G(\sigma_{\mathcal{D}_N}) = 20\log_{10}\left((1+N+\sqrt{N(N+2)}\right)$$





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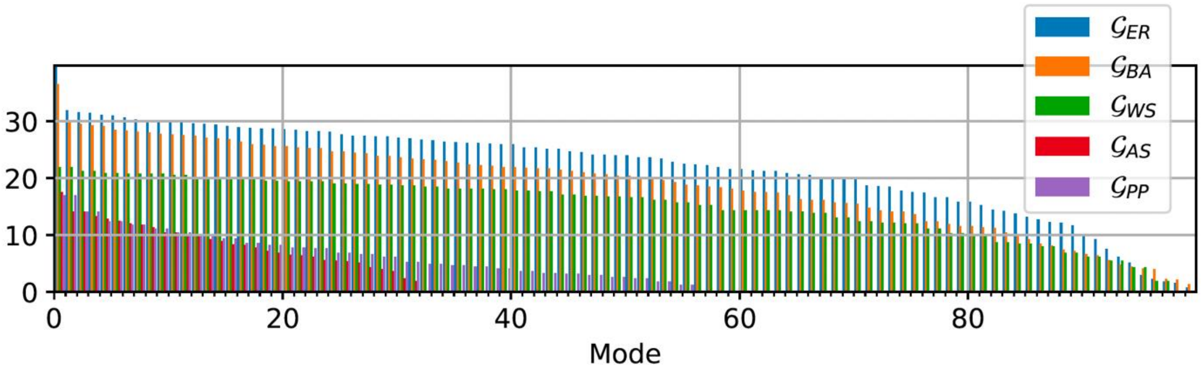

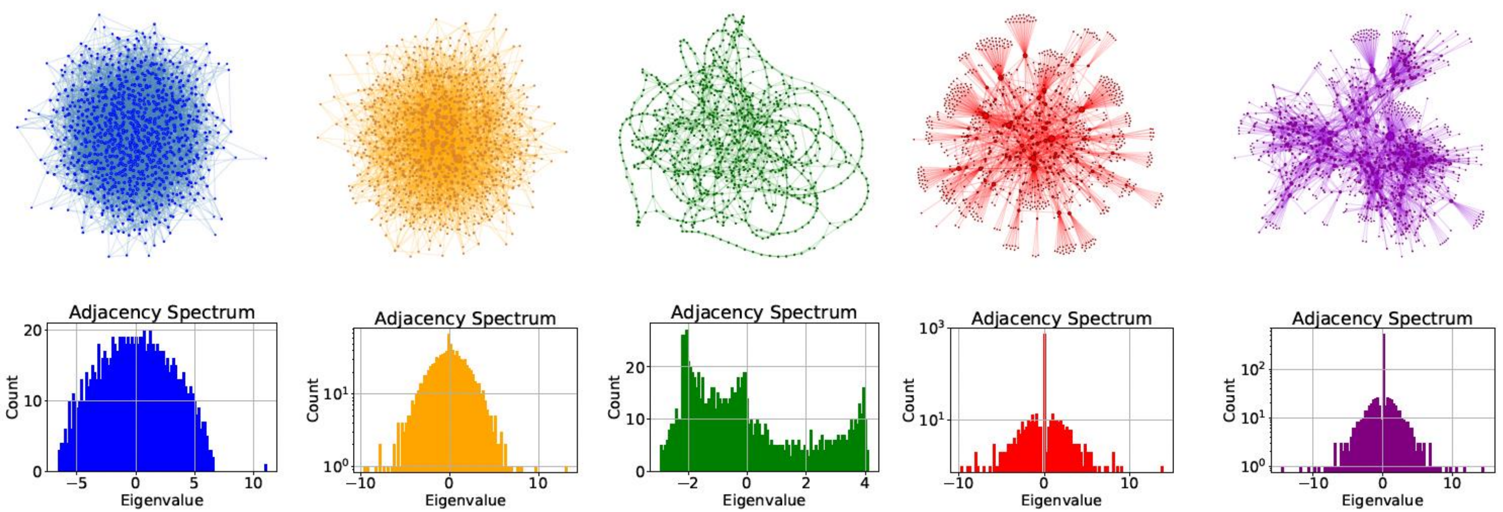

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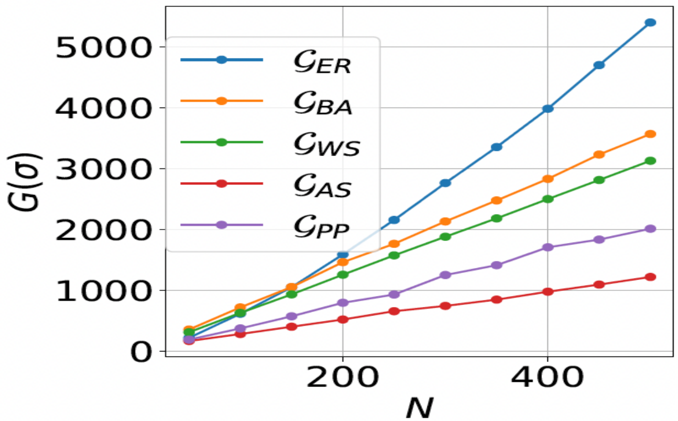
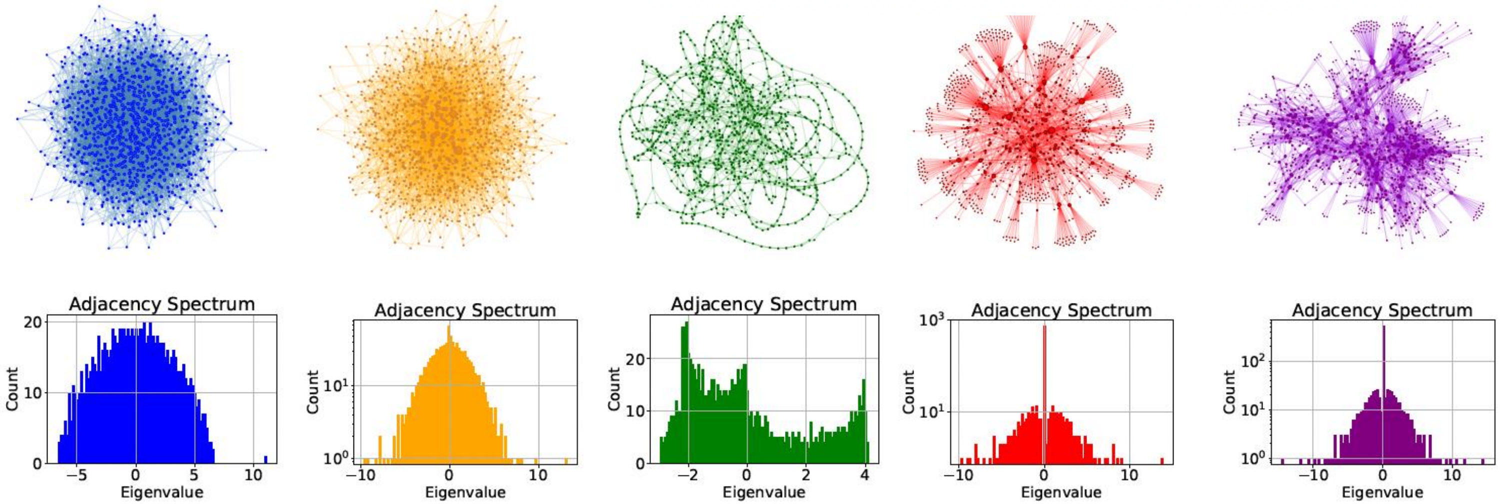
In this case, the networks were simulated and the values obtained were averaged over ten different samples.



Mainly linear except for the ER (super-linear)

Dominant term

$$= 10N \log_{10}(Np(1-p)) + O(N)$$



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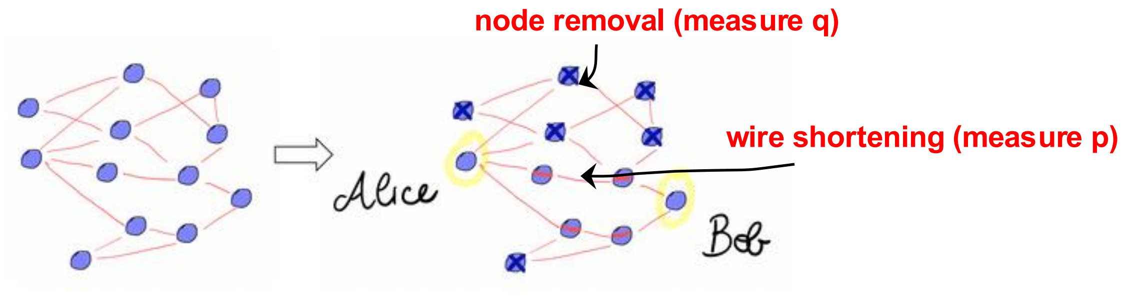

Routing : local homodyne measurements
(node removal or wire shortening)

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Cost and routing of a graph (cluster)

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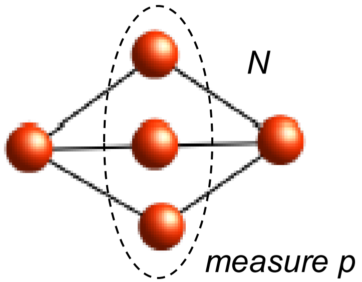
$\mathcal{D}_5$:



$$\mathcal{N}^{(\mathcal{D}_N)} = \log_2(1 + 2N Rg^2)$$

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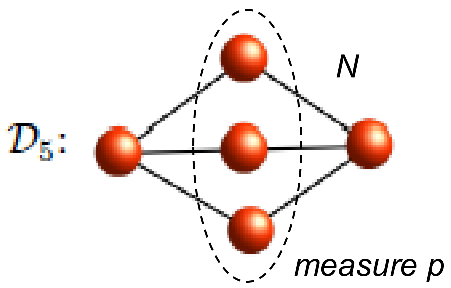

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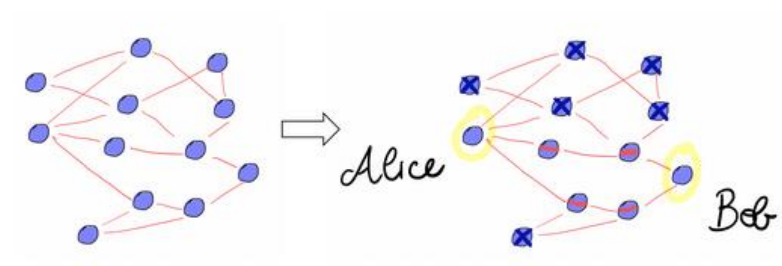

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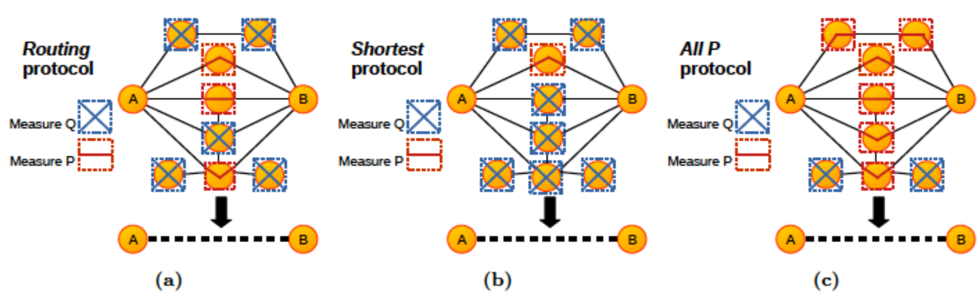

Routing protocol with parallel paths enhancement

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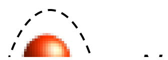
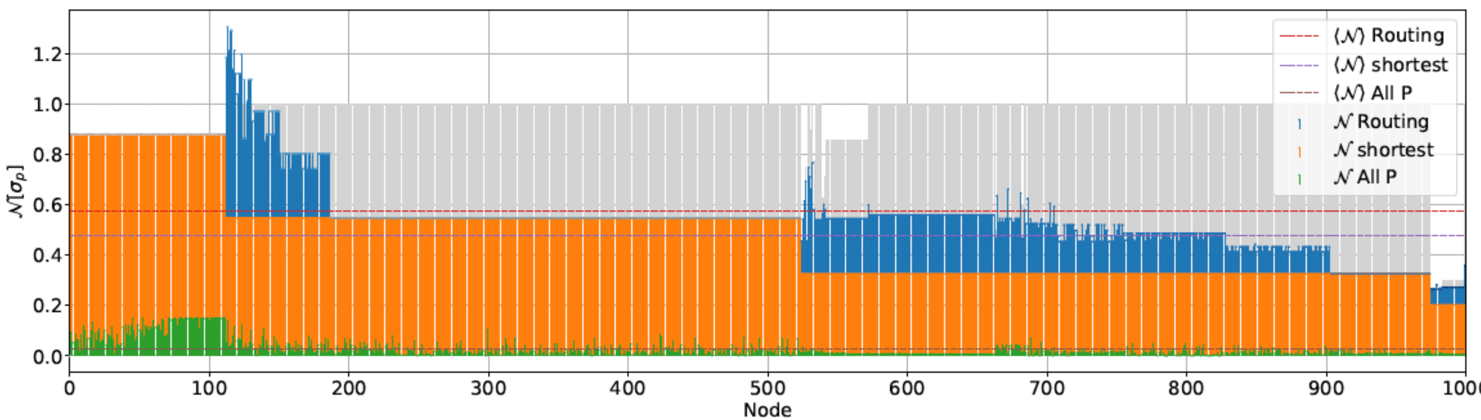


Via measurements



ous System

FIG. 7: Logarithmic negativity produced by the three different protocols applied to each node of the the Autonomous System $\mathcal{G}_{AS}(N = 1000)$ network. The nodes are labeled in order of distance and of number of paths connecting to Alice. The blue, orange and green stems represent the logarithmic negativity of the final pair after the *Routing*, *Shortest* and *All P* protocols respectively, while the dashed lines represent the mean value for all the nodes. The color of the marker indicates the distance of the node from A and the grey columns represent the ratio of paths that improved the entanglement in *Routing*.

***Routing* protoco**

Measure Q

Measure P

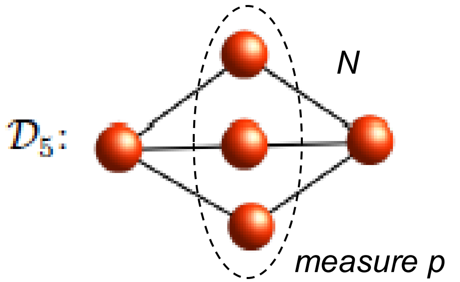
Routing protocol with parallel paths enhancement

F. Centrone

F. Centrone *et al.* Physical Review A 108 (4), 042615 (2023)

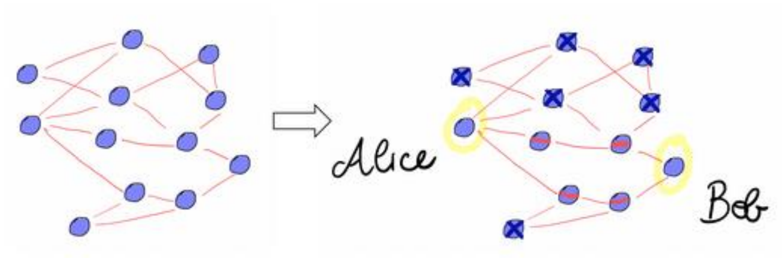
Cost and routing of a graph (cluster)

Idea: in the CV case is easier to prepare large entanglement networks and then reshape

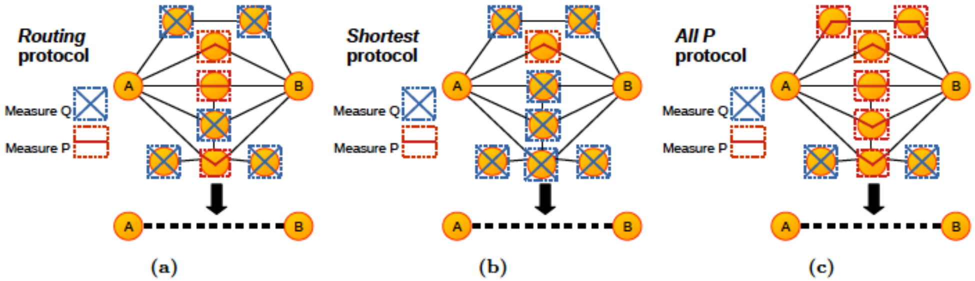


$$\mathcal{N}^{(\mathcal{D}_N)} = \log_2(1 + 2N\,Rg^2)$$

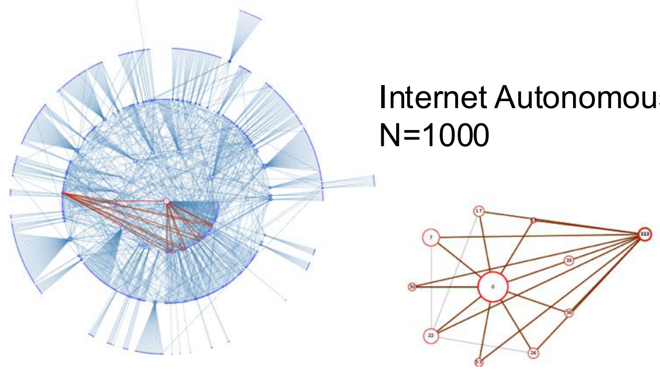
Via measurements



Routing : local homodyne measurements
(node removal or wire shortening)



Routing protocol with parallel paths enhancement



Internet Autonomous System
N=1000

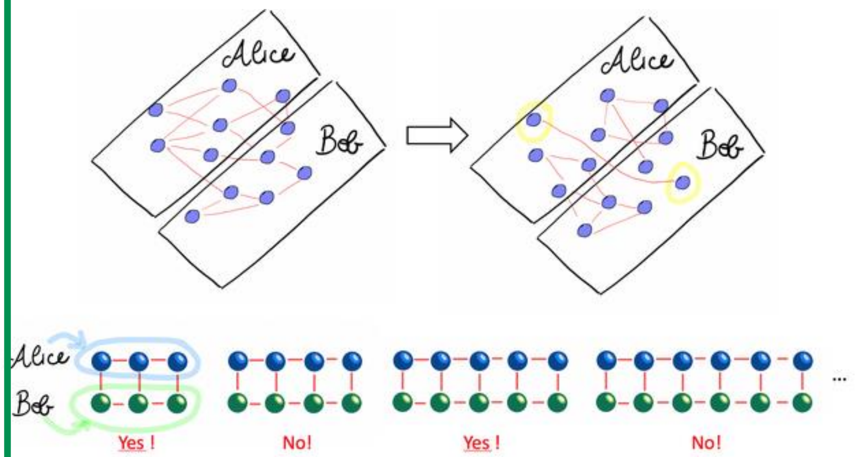
F. Centrone

F. Centrone *et al.* Physical Review A 108 (4), 042615 (2023)

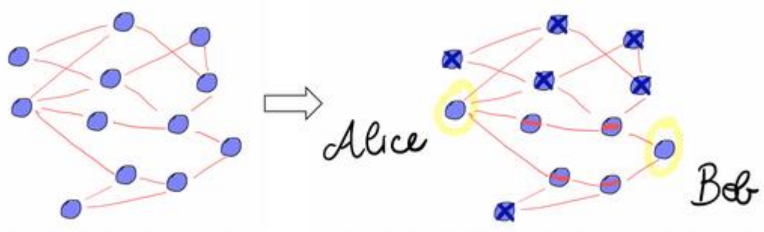
Routing of a graph via linear optics

Idea: in the CV case is easier to prepare large entanglement networks and then reshape

Via linear optics



Via measurements



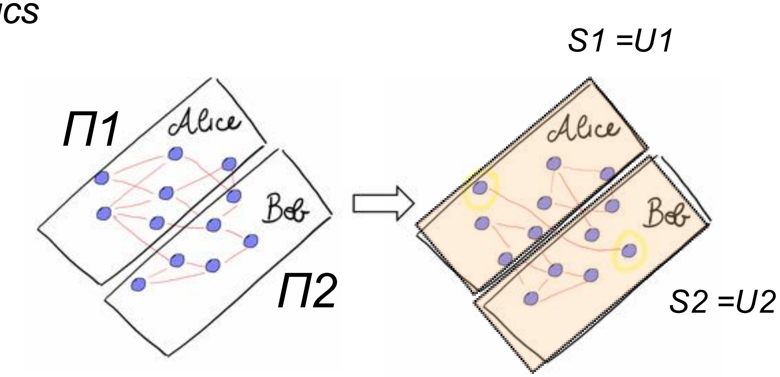
F. Sansavini and V. Parigi, *Entropy* 22, 26 (2019)
D. Fainsin et al. arXiv:2503.20547 (2025)

F. Centrone *et al.* Physical Review A 108 (4), 042615 (2023)

Routing of a graph via linear optics

Idea: in the CV case is easier to prepare large entanglement networks and then reshape

Via linear optics



We have two providers, Π1 and Π2, each control half of a quantum network of arbitrary size n.

These providers serve numerous clients who wish to communicate securely.

Two clients Alive and Bob want to communicate by sharing the best entanglement connection, either they belong to the same provider or not

We focus here on less costly operations that can be implemented by the providers, i.e. passive unitaries (multiport interferometers)

(in case of general symplectic transformation the problem has been solved)

F. Sansavini and V. Parigi, *Entropy 22*, 26 (2019)

D. Fainsin, A. Debray, I. Karuseichyk, M. Walschaers, V. Parigi. ***Entanglement routing via passive optics in CV-networks arXiv:2503.20547 (2025)***

Routing of a graph via linear optics

Covariance matrix Γ_G $\qquad \Gamma_{Gij} = (1/2)\langle\hat{\xi}_i\hat{\xi}_j + \hat{\xi}_j\hat{\xi}_i\rangle$

$$\hat{\xi}^T = (\hat{Q}, \hat{P})^T$$

$p_{i,j}$ *or* $q_{i,j}$

The ideal state we want to reach for Alice and Bob is the two node-cluster (close to the two-mode squeezed state) with covariace matrix

$$\Gamma_{\bullet\!-\!\bullet} = \begin{pmatrix} \lambda & 0 & 0 & \mu \\ 0 & \lambda & \mu & 0 \\ 0 & \mu & \lambda & 0 \\ \mu & 0 & 0 & \lambda \end{pmatrix} \qquad \begin{aligned} \mu &= \sinh(2r) \\ \lambda &= \cosh(2r) \end{aligned}$$

$\Downarrow$

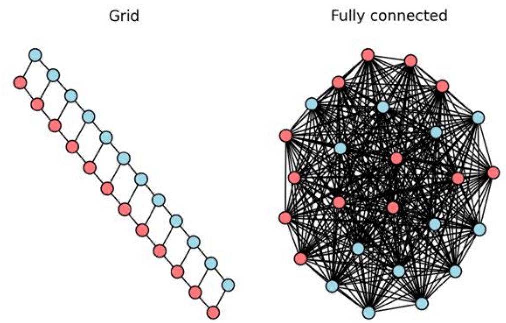
Ideal routing result

Imperfect routing: The two nodes are entangled but less than above, they are still disconnected from the rest

The way we look for the result: evolutionary algorithm maximizing fitness function on covariance matrix. We follow a derandomized evolution strategy (DES) to find the appropriate unitaries (U1U2) (with Gell-Mann parametrization)

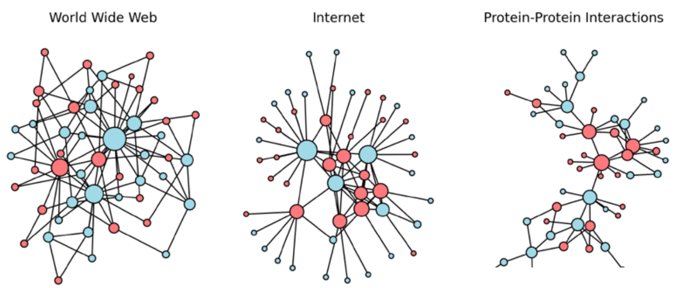
Routing of a graph via linear optics

Results



Regular/ deterministic networks (up to 100 nodes)

Topology	Even Grid	Odd Grid	Fully Conn.
Routing (Alice/Bob)	No	Yes	No
Routing (Alice/Alice)	No	No	Yes



Complex network (probabilistic) models

BA = Barabasi Albert- WWW
AS = Internet as a graph model
PP= duplication-divergence

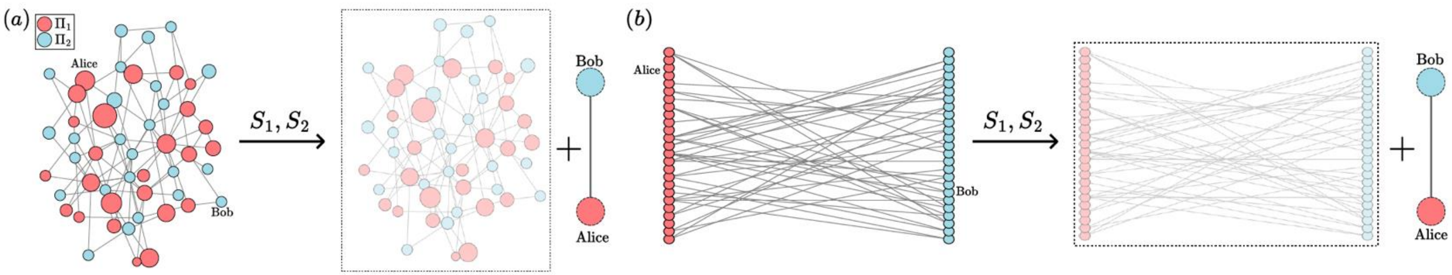
100 simulations

	BA			AS			PP		
Scenario	I	II	III	I	II	III	I	II	III
Purity $\bar{\gamma}$ (%)	97.49	98.43	98.83	98.97	99.54	99.34	98.15	98.45	98.04
std(γ) (%)	4.07	2.06	1.52	2.05	0.8	0.78	2.9	2.02	2.55
$\bar{f}_{opt}$	0.64	0.62	0.59	0.7	0.71	0.74	0.6	0.63	0.62
std(f_{opt})	0.27	0.23	0.21	0.38	0.48	0.45	0.33	0.32	0.34

I Alice is a hub node and Bob a node with low degree;
II Alice and Bob are two nodes with maximum distance in the graph;
III Alice and Bob are two nodes with low degrees.

Routing of a graph via linear optics

In light of the results, most quantum networks with complex shapes derived from real-world networks are compatible with a bipartite entanglement routing task.



Routing in a Barabasi-Albert network
a) Red nodes: belong to Alice, blue nodes to Bob.
In the initial figure, the dimensions of each node are directly proportional to its degree, i.e. the number of links that point toward it. After the routing Alice and Bob get the EPR state. The opacity on the right-hand side of both figures signifies that the resulting n − 2 state does not necessarily constitute a graph state, no stipulations are made regarding the precise form of this state.
b) Bipartite layout of the procedure

Routing of a graph via linear optics

LKB

No-go criteria: the symplectic distribution of deterministic and complex topologies

If we consider 2 local symplectic transformations SA and SB applied by each procider, they correspond to the Williamson decompositions of their local state_> then the resulting state is locally equivalent to a set of thermal states.
An evaluation of the symplectic eigenvalues of the local states allows us to derive two no-go criteria for the routing problem.

Criteria 1 (*Entanglement Routing*). *Let $\sigma(\Gamma_W)$ be the spectrum of Γ_W. The spectrum $\sigma(\Gamma_W)$ is conserved under all symplectic transformation, which include passive transformations. We can thus state, if $\lambda \notin \sigma(\Gamma_W)$, then* ***ideal routing*** *is impossible.*

Williamson decomposition of the covariance matrix

$$\Gamma_{j,k} = (1/2)\langle \hat{\xi}_i \hat{\xi}_j + \hat{\xi}_j \hat{\xi}_i \rangle \qquad \Gamma = S.\sigma S^T$$

$$\sigma = \mathrm{diag}\,(d_1, d_2, \ldots, d_n, d_1, \ldots, d_n) \qquad d_i \geq \sigma_0 \longrightarrow$$

Vacuum or thermal states

S *Symplectic* $\quad S = R_1 \Delta^{sq} R_2$

Routing of a graph via linear optics

No-go criteria: the symplectic distribution of deterministic and complex topologies

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Criteria 2 (*Internal Entanglement Routing*). *In the case of internal routing a necessary condition for **ideal routing** is to have the value 1 with multiplicity 2 in the symplectic spectrum $\sigma(\Gamma_W)$.*

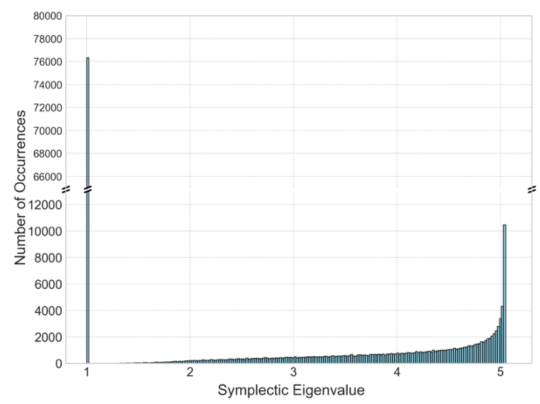

FIG. 5: **Histogram of symplectic eigenvalues from 100 Barabasi-Albert graphs.** Each graph was composed of 1000 nodes equally splitted between two users leading to 200 000 symplectic values. The initial squeezing parameter is arbitrary chosen to be $s = 10$ such that $\lambda = 5.05$.

Routing of a graph via linear optics

$$\Gamma = S\sigma S^T$$

$$S = R_1 \Delta^{sq} R_2$$

$$\Gamma = S\sigma S^T = R_1 \Delta^{sq} R_2 \sigma R_2^T \Delta^{sq} R_1^T$$

σ represents a set of thermal states, with variance are greater or equal to that of the vacuum, it shows no logarithmic negativity, $R_2 \sigma R_2^T$ and Δ^{sq} creates no quantum correlations. Only R_1 is responsible foo the entanglement.
So when the state is built from equally squeezed state, the passive transformation $U = diag(R_1, R_2)$ is a solution for the routing problem, i.e. $U^T \Gamma U$ is a perfected routed state if $\lambda \in \sigma(\Gamma_W)$.